CHAPTER (4)

FLOWING FLUIDS AND PRESSURE VARIATION

SUMMARY

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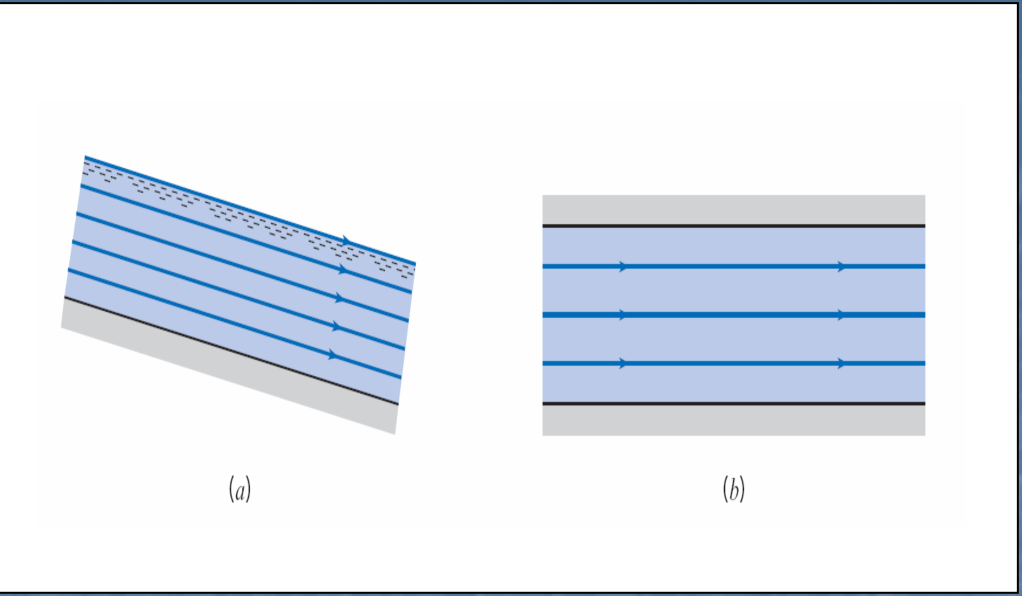
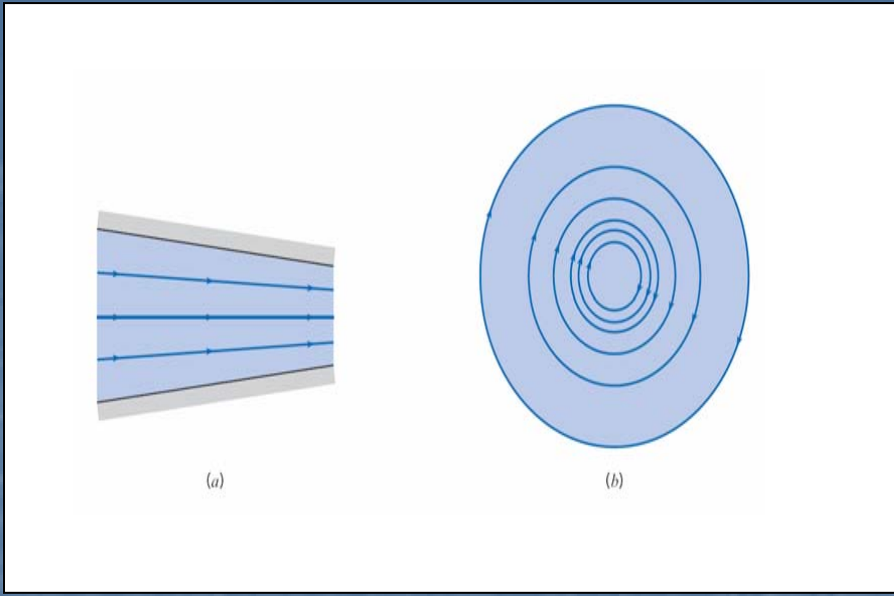




Uniform Flow

$$\frac{\partial V}{\partial s} = 0$$

Non-Uniform Flow

$$\frac{\partial V}{\partial s} \neq 0$$

Steady Flow

$$\frac{\partial V}{dt} = 0$$

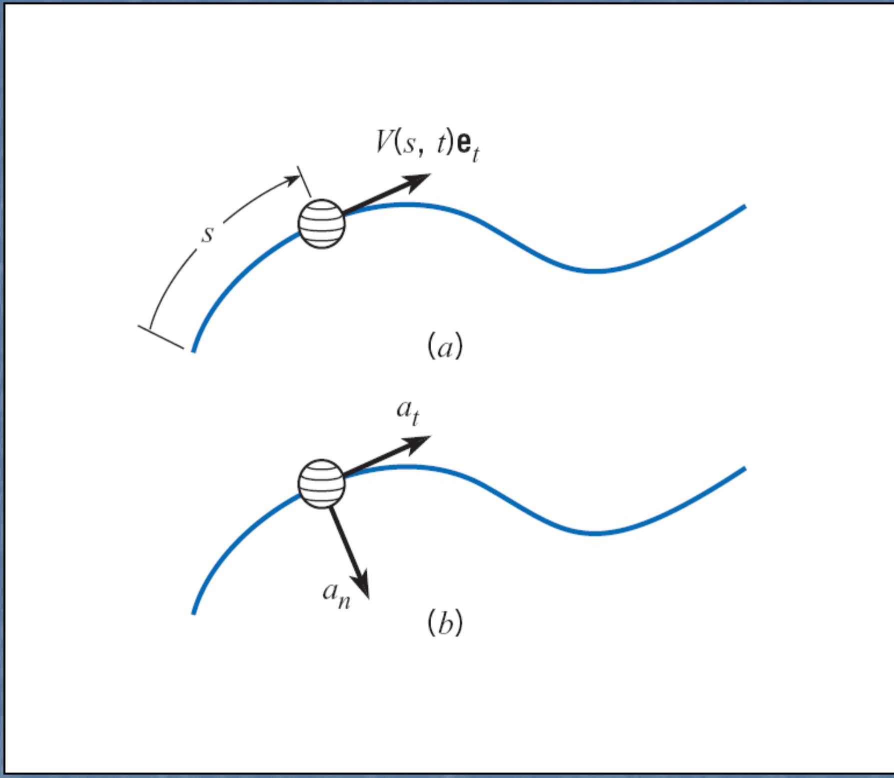
Non-Steady Flow

$$\frac{\partial V}{dt} \neq 0$$

Fluid Motion

The acceleration of fluid particle can be expressed as

$$a=\frac{dV}{dt}=\left(\frac{dV}{dt}\right)e_t+V\left(\frac{de_t}{dt}\right)$$



Where $$\left(\frac{dV}{dt}\right)e_t=\frac{dV(s,t)}{dt}=\left(\frac{\partial V}{\partial s}\right)\left(\frac{\partial s}{\partial t}\right)+\left(\frac{\partial V}{\partial t}\right)$$

$$\frac{dV}{dt}=V\left(\frac{\partial V}{\partial s}\right)+\left(\frac{\partial V}{\partial t}\right)$$

and $$\left(\frac{de_t}{dt}\right)=\left(\frac{V}{r}\right)e_n$$

Where:

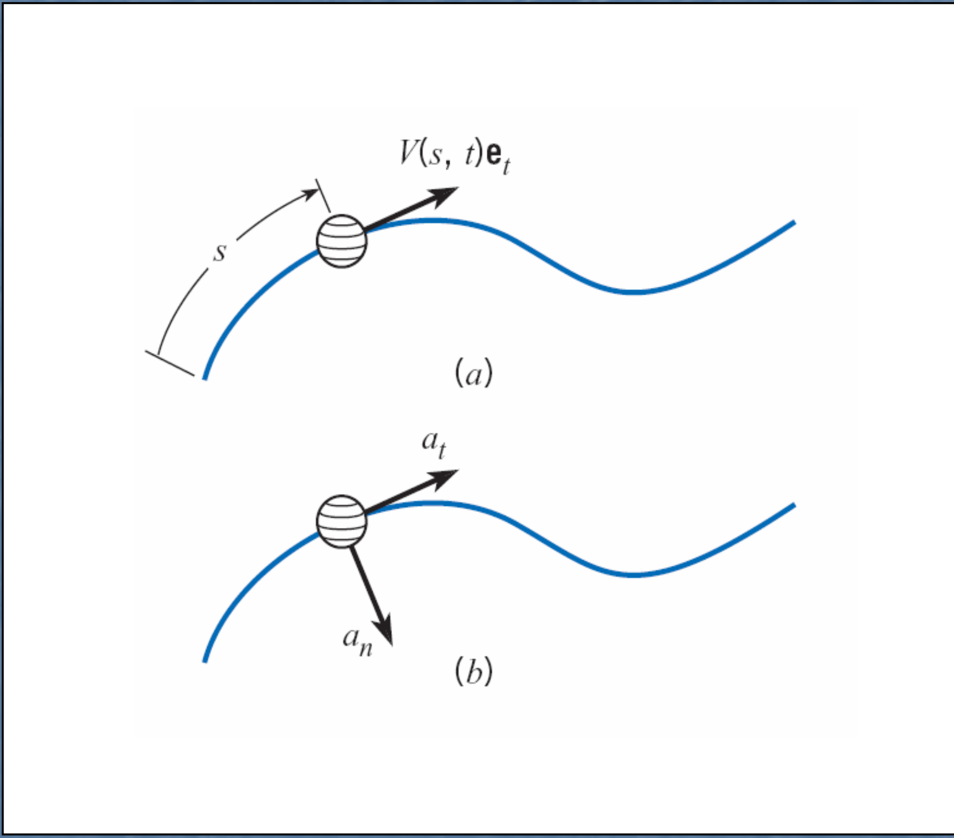
r = radius of local curvature

e_n = unit vector that is perpendicular to the pathline

Fluid Motion

Substituting the values (previous slide) in the acceleration Eqn, we have

$$a = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)e_t + \left(\frac{V^2}{r}\right)e_n$$



Where: $a_t = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)$ $a_n = \left(\frac{V^2}{r}\right)$

i.e $a = a_t + a_n$

$a_t = Tangential\ acceleration$

$a_n = Normal\ acceleration$

Fluid Motion

The acceleration in the Z-direction is given by,

$$a_z = \frac{dw}{dt} = \left(\frac{\partial w}{\partial x}\frac{dx}{dt}\right) + \left(\frac{\partial w}{\partial y}\frac{dy}{\partial t}\right) + \left(\frac{\partial w}{\partial z}\frac{dz}{\partial t}\right) + \left(\frac{\partial w}{\partial t}\right)$$

As $u = \left(\frac{dx}{dt}\right)$ $v = \left(\frac{dy}{dt}\right)$ $w = \left(\frac{dz}{dt}\right)$

The acceleration in the X-direction: $a_x = \frac{du}{dt} = \left(u\frac{\partial u}{\partial x}\right) + \left(v\frac{\partial u}{\partial y}\right) + \left(w\frac{\partial u}{\partial z}\right) + \left(\frac{\partial u}{\partial t}\right)$

The acceleration in the Y-direction: $a_y = \frac{dy}{dt} = \left(u\frac{\partial v}{\partial x}\right) + \left(v\frac{\partial v}{\partial y}\right) + \left(w\frac{\partial v}{\partial z}\right) + \left(\frac{\partial v}{\partial t}\right)$

The acceleration in the Z-direction: $a_z = \frac{du}{dt} = \left(u\frac{\partial w}{\partial x}\right) + \left(v\frac{\partial w}{\partial y}\right) + \left(w\frac{\partial w}{\partial z}\right) + \left(\frac{\partial w}{\partial t}\right)$

Example (4.1)

The velocity field for a fluid is given by

$$\mathbf{V} = 2x^2t\mathbf{i} + 3xy^2\mathbf{j} + 2xz\mathbf{k}$$

Find the acceleration in the x direction at the point (1,2,2) when $t = 1$. The coefficients in the equation have dimensions such that when the position is expressed in meters and time in seconds, the velocity is in m/s.

Solution The acceleration in the x direction is given by

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

The velocity components are

$$u = 2x^2t$$

$$V = ui + vj + wk \qquad v = 3xy^2$$

$$w = 2xz$$

Carrying out the derivatives of the velocities and substituting into Eq. (4.11) gives

$$a_x = 2x^2 + 2x^2t \times 4xt + 3x^2y \times 0 + 2xz \times 0$$

Substituting in the values and carrying out the calculations gives

$$a_x = 2 + 8 = 10 \text{ m/s}^2$$

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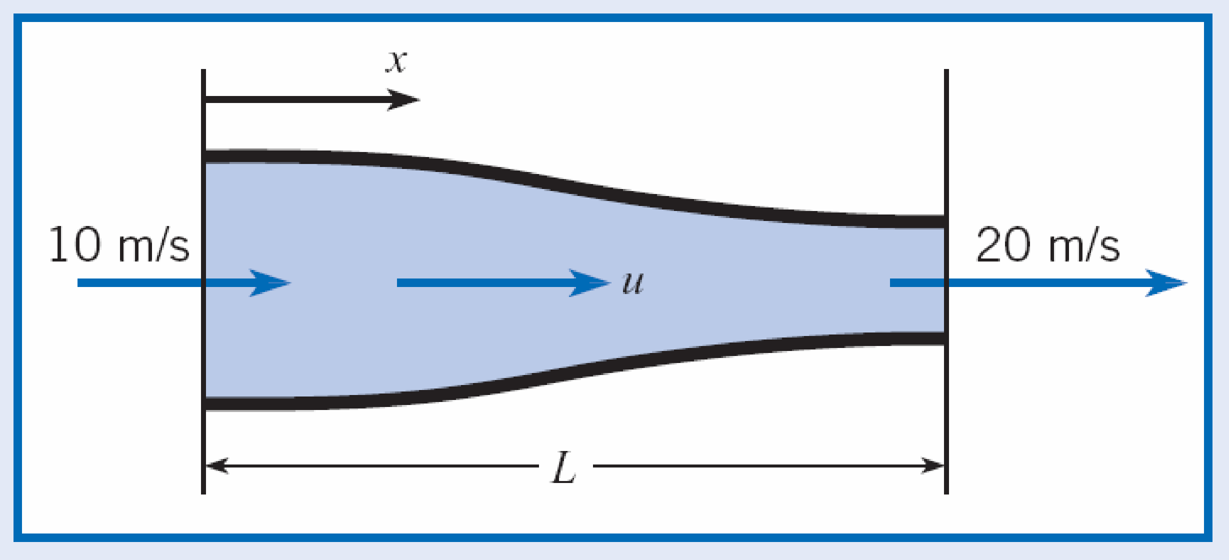
Equations (4.12) and (4.13) can be used in the same fashion to find the components of acceleration in the y and z directions.

Example (4.2)

A nozzle is designed such that the velocity in the nozzle varies as

$$u = \frac{u_0}{1.0 - 0.5x/L}$$

where the velocity u_0 is the entrance velocity and L is the nozzle length. The entrance velocity is 10 m/s and the length is 0.5 m. The velocity is uniform across each section. Find the acceleration at the station halfway through the nozzle $(x/L = 0.5)$.



$$a_x = \frac{du}{dt} = \left(u\frac{\partial u}{\partial x}\right) + \left(v\frac{\partial u}{\partial y}\right) + \left(w\frac{\partial u}{\partial z}\right) + \left(\frac{\partial u}{\partial t}\right)$$

$$u\frac{\partial u}{\partial x}$$

$$a_x = u\frac{du}{dx}$$

where the total derivative has been used because u is a function of x only.
Taking the derivative

$$\frac{du}{dx} = -\frac{u_0}{(1-0.5x/L)^2}\times\left(-\frac{0.5}{L}\right)$$
$$= \frac{1}{L}\frac{0.5u_0}{(1-0.5x/L)^2}$$

and multiplying by u gives

$$u\frac{du}{dx} = 0.5\frac{u_0^2}{L}\frac{1}{(1-0.5x/L)^3}$$

Evaluating the acceleration at $x/L = 0.5$ gives

$$a_x = 1.185\frac{u_0^2}{L}$$
$$= 1.185\times\frac{10^2}{0.5}$$
$$= 237\ \text{m/s}^2$$

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Since a_x is positive, the direction of the acceleration is positive, that is, in the positive x direction. This is reasonable because the velocity increases in the x direction.

Fluid Motion

$$\sum F_l = ma_l = p\Delta A - (p + \Delta p)\Delta A - \Delta W \sin\alpha$$

$$\rho V a_l = p\Delta A - p\Delta A - \Delta p \Delta A - \rho V g \sin\alpha$$

Re-Arranging

$$\rho(\Delta A \Delta l) a_l = p\Delta A - p\Delta A - \Delta p \Delta A - \rho(\Delta A \Delta l) g \sin\alpha$$

$$\rho(\Delta l) a_l = -\Delta p - \rho(\Delta l) g \sin\alpha$$

$$\rho a_l = -\frac{\Delta p}{\Delta l} - \rho g \sin\alpha$$

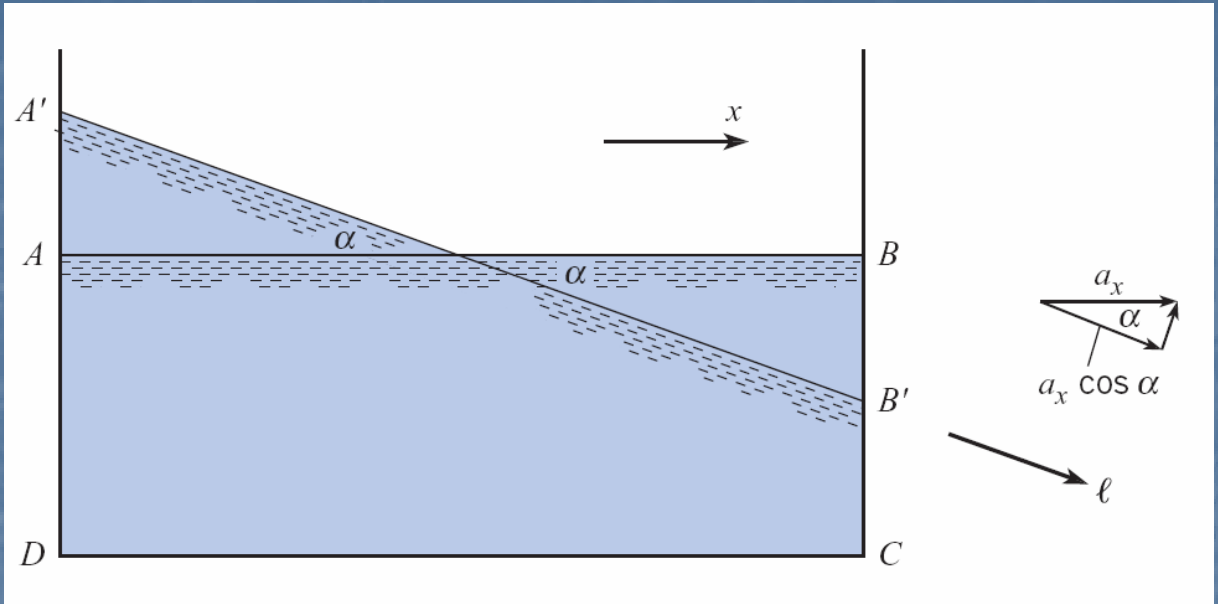
$\sin\alpha = \frac{dz}{dl}$ $(\gamma = \rho g)$ $\Delta l \to 0$

$$-\frac{dp}{dl} - \gamma \frac{dz}{dl} = \rho a_l$$

Euler Equation of motion

$$-\frac{d}{dl}(p + \gamma z) = \rho a_l$$

Uniform acceleration of liquid in a tank



Applying Euler formula along $A'B'$

$$-\frac{d}{dl}(p+\gamma z) = \rho a_l$$

$$-\frac{dp}{dl} - \gamma\frac{dp}{dz} = \rho a_l$$

$$\frac{dp}{dl} = 0 \text{ as the change in pressure is zero}$$

Surface is open to atmosphere

$$-\frac{d}{dl}(\gamma z) = \rho a_l$$

$$\frac{dz}{dl} = \frac{a_l}{g} = \frac{a_x \cos\alpha}{g}$$

Fluid Motion

But $\frac{dz}{dl} = -\sin\alpha$ Then $\sin\alpha = \frac{a_x \cos\alpha}{g}$ Hence $\tan\alpha = \frac{a_x}{g}$

Acceleration of liquid in X-direction $a_x = g\tan\alpha$

Still further analysis can be made if Euler's equation is applied along a horizontal plane in the liquid, such as at the bottom of the tank. Now z is constant and Euler's equation reduces to $\partial p/\partial \ell = -\rho a_x$, which shows that the pressure must decrease in the direction of acceleration. The change in pressure is consistent with the change in depth of the liquid because hydrostatic pressure variation prevails in the vertical direction, since there is no component of acceleration in that direction. Thus as the depth decreases in the direction of acceleration, the pressure along the bottom of the tank must also decrease. Another case of uniform acceleration is given in the following example.

Fluid Motion

$$-\frac{d}{ds}\left(p+\gamma z+\rho\frac{V^2}{2}\right)=0$$

$$\left(p+\gamma z+\rho\frac{V^2}{2}\right)=C$$

Equation above is called the Bernoulli's Equation which states that

The sum of piezometric pressure $(p+\gamma z)$ and the kinetic or dynamic pressure $(\rho\frac{V^2}{2})$ is equal constant for a steady, incompressible, inviscid fluid,

Another form of Bernoulli's Equation can be expressed as follows,

$$\left(\frac{p}{\gamma}+z+\frac{V^2}{2g}\right)=\left(h+\frac{V^2}{2g}\right)=C$$

Where

$$h=Piezometric\ head \quad and \quad \left(\frac{V^2}{2g}\right)=Dynamic\ head$$

Stagnation Tube

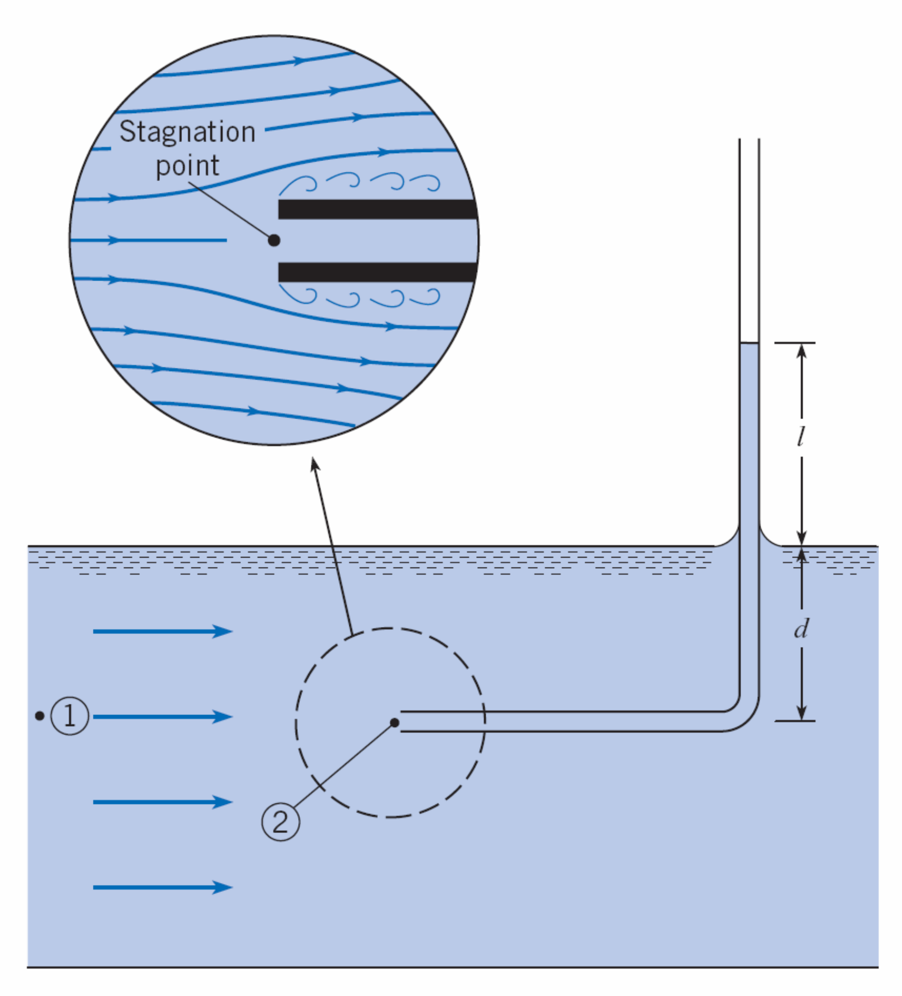
Applying Bernoulli's Eqn. between points (1) and (2), we have,

$$\left(p_1 + \gamma z_1 + \rho \frac{V_1^{\,2}}{2}\right) = \left(p_2 + \gamma z_2 + \rho \frac{V_2^{\,2}}{2}\right)$$

$$z_1 = z_2$$

$$\left(p_1 + \rho \frac{V_1^{\,2}}{2}\right) = \left(p_2 + \rho \frac{V_2^{\,2}}{2}\right)$$

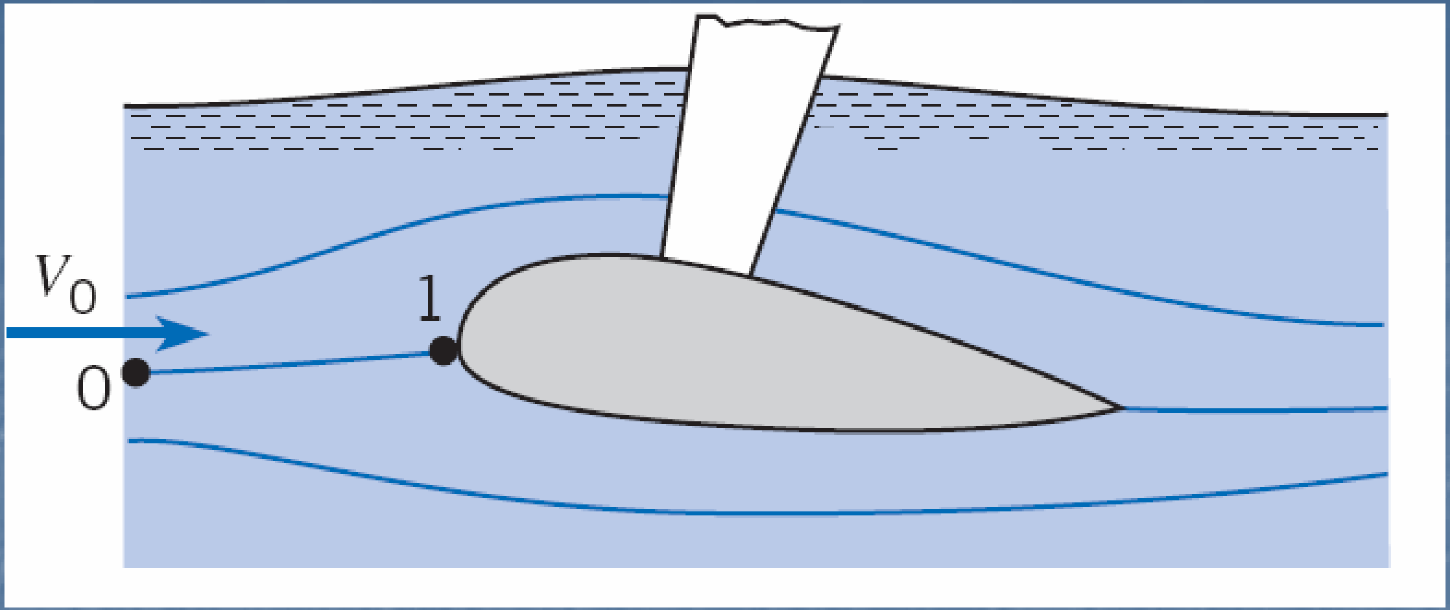
Since $V_2 = 0$ (a stagnation point)



$$\left(\frac{V_1^{\,2}}{2}\right) = \frac{2}{\rho}(p_2 - p_1)$$

$$p_2 = \gamma(l + d) \quad and \quad p_1 = \gamma d$$

$$V_1 = \sqrt{2gl}$$



By applying the Bernoulli equation between points 0 & 1, we have,

$$h_0 + \frac{V_0^2}{2g} = h_1 + \frac{V_1^2}{2g}$$

$$V_1 = 0$$

$$h_1 - h_0 = \frac{V_0^2}{2g}$$

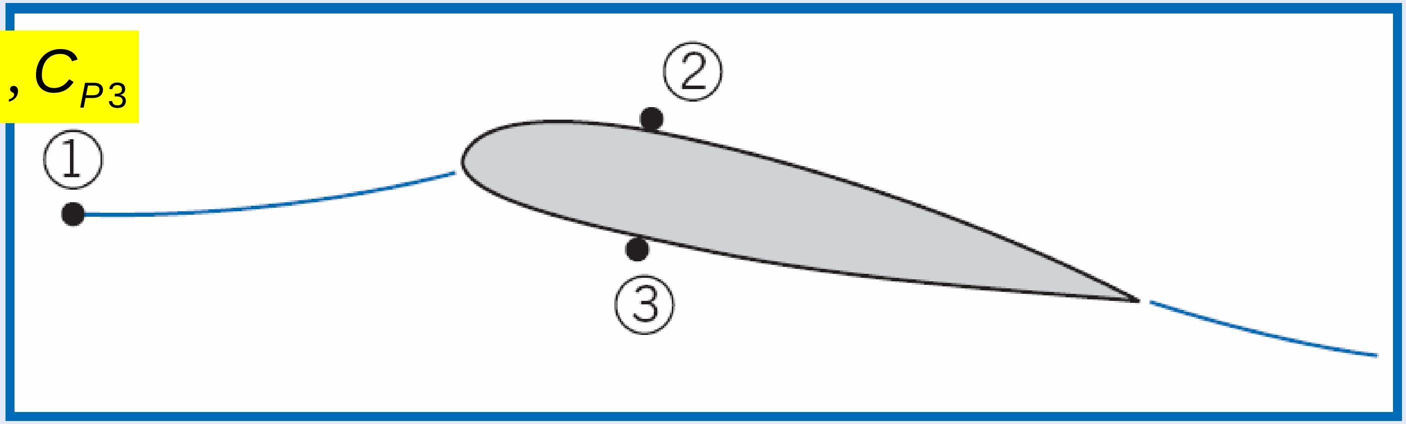
$$C_p = \frac{h - h_0}{V_0^2 / 2g}$$

i.e

$$C_p = 1$$

Example (4.6)

Find $(p_2 - p_3), C_{P2}, C_{P3}$



$V_1 = 300\, ft/s,\ p_1 = 14\, psia$

$V_2 = 330\, ft/s, V_3 = 270\, ft/s$

Applying Bernoulli's equation between 1 & 2, we have $p_1 + \rho\frac{V_1^{\,2}}{2} = p_2 + \rho\frac{V_2^{\,2}}{2}$

Applying Bernoulli's equation between 1 & 3, we have $p_1 + \rho\frac{V_1^{\,2}}{2} = p_3 + \rho\frac{V_3^{\,2}}{2}$

$p_2 + \rho\frac{V_2^{\,2}}{2} = p_3 + \rho\frac{V_3^{\,2}}{2}$ $\quad p_3 - p_2 = \frac{\rho}{2}\left(V_2^{\,2} - V_3^{\,2}\right)$

$p_1 + \rho\frac{V_1^{\,2}}{2} = p_2 + \rho\frac{V_2^{\,2}}{2}$ $\quad p_2 - p_1 = \frac{\rho}{2}\left(V_1^{\,2} - V_2^{\,2}\right)$

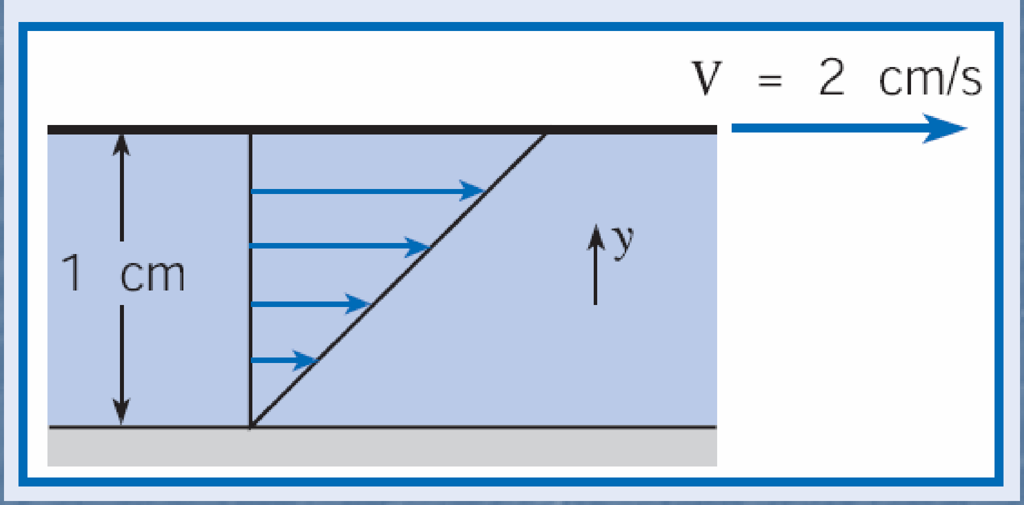
But $C_{p2} = \dfrac{p_2 - p_1}{\frac{1}{2}\rho V_1^{\,2}}$ $\quad C_p = \dfrac{p - p_0}{\frac{1}{2}\rho V_0^{\,2}}$

$C_{p_2} = 1 - \left(\frac{V_2}{V_1}\right)^2$ Same way $C_{p_3} = 1 - \left(\frac{V_3}{V_1}\right)^2$

$C_{p2} = -0.21$ $\quad C_{p3} = 0.19$

Example (4.8)

A fluid exists between stationary and moving parallel flat plates, and the velocity is linear shown. The distance between the plates is 1 cm and the upper plate moves at 2 cm/s. Fir the amount of rotation that fluid elements located at 0.25 cm, 0.5 cm, and 0.75 cm will underg after they have traveled a distance of 1 cm.



Fluid Motion

$\Omega_z = \omega$ **For rotational flow Or Forced Vortex**

For irrotational flow $\Omega_z = 0$

$$\dot{\theta} = \Omega_z = \frac{1}{2}\left(\frac{dV}{dr} + \frac{V}{r}\right)$$

$$\frac{dV}{dr} = -\frac{V}{r}$$

$$\frac{dV}{V} = -\frac{dr}{r}$$

$V = \frac{C}{r}$ **or** $Vr = C$ **Free Vortex Flow**

In Free Vortex Flow, the Tangential Velocity Varies Inversely with Radius

For liquid rotating as a rigid body, i.e. $V = \omega r$

$$\frac{d}{dr}\left(p + \gamma z\right) = \rho r \omega^2$$

Integrating the above equation

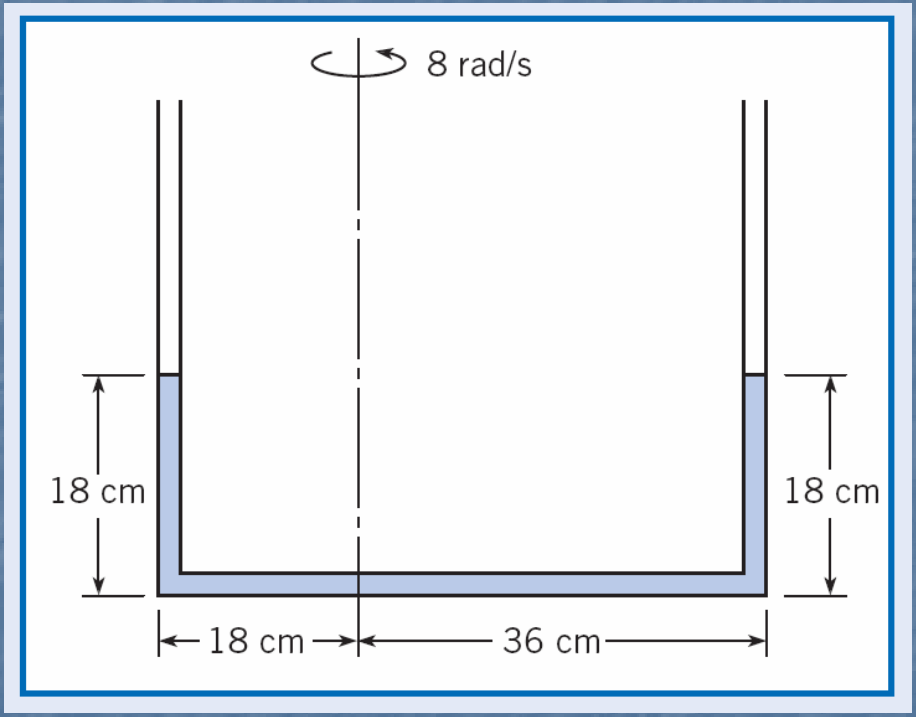
$$\left(p + \gamma z\right) = \frac{\rho r^2 \omega^2}{2} + C$$

$$\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g}\right) = C$$

<u>**The above equation describes the pressure variation in rotating flow**</u>

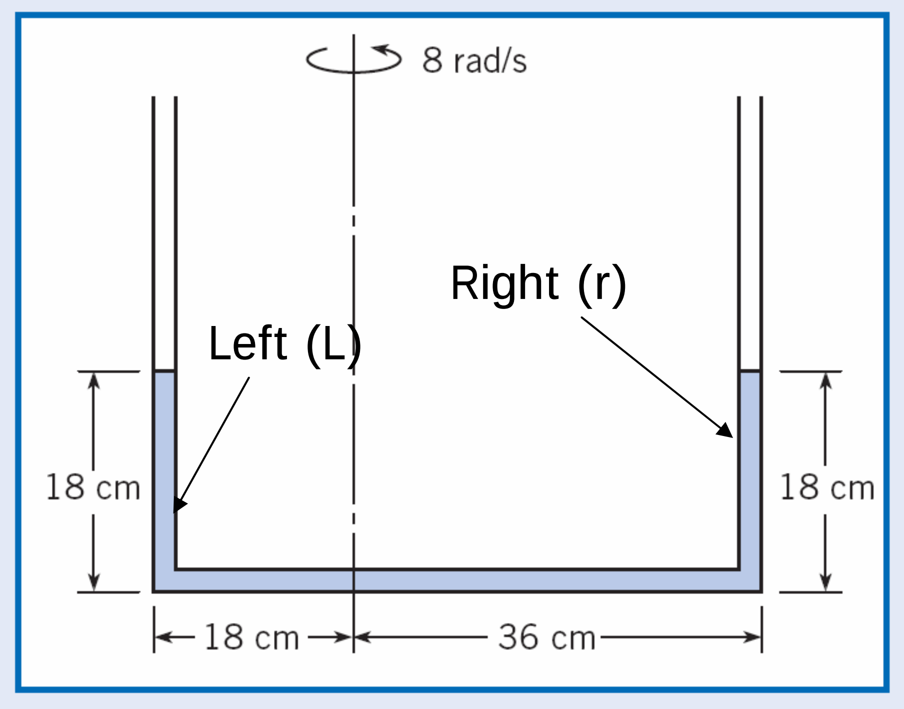
Example (4.10)

When the U-tube is not rotated, the water stands in the tube as shown. If the tube is ro tated about the eccentric axis at a rate of 8 rad/s, what are the new levels of water in th tube?



Find the new level of water during rotation?

Applying $\left(\frac{p}{\gamma}+z-\frac{\omega^2 r^2}{2g}\right)=C$ about the line of rotation



P=0 as atmospheric, then

$$\gamma z_l-\left(\frac{\rho(r_l\omega)^2}{2}\right)=\gamma z_r-\left(\frac{\rho(r_r\omega)^2}{2}\right)$$

$$z_l-z_r=\left(\frac{\omega^2}{2g}\right)\left(r_l^{\,2}-r_r^{\,2}\right)$$

$$z_l+z_r=0.36$$

$$z_l=2.1cm \quad z_r=33.9\,cm$$

END SUMMARY

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